# Stress Analysis of Compression of Aluminium with Rotating Tools

X. Ma<sup>1</sup>, M.R. Barnett<sup>1</sup>, Y.H. Kim<sup>2</sup>

<sup>1</sup> School of Engineering and Technology, Deakin University, Geelong, VIC 3217, Australia <sup>2</sup> ERC/NSDM, School of Mechanical Engineering, Pusan National University, South Korea

Keywords: Compression; Die Rotation; Stress Analysis; Load Reduction.

# Abstract

Compression tests carried out on aluminium specimens showed that when the die was rotated the compression load dropped. A slab method is employed to examine this process. The load reduction is explained by the deviation of friction vector due to the relative circumferential movement between the die and the material. This mechanism is incorporated into a theoretical model and an expression is derived for compression pressure. Analytical solutions established compare favourably with experimental results. It is also shown that there is a limitation to the load reduction: the compressive load can never be lower than 70 percent of the yield limit.

# 1. Introduction

Compression or upsetting of cylindrical workpieces is one of most fundamental operations in the metal forming industry. It is well known that during this process interfacial friction leads to the development of inhomogeneous plastic deformation inside the material [1]. A higher frictional resistance demands larger forming loads and tends to shorten tool life. A number of methods have been developed to reduce or eliminate these negative effects of friction. Siebel and Pomp [2] suggested using conically convex platens which matched conical cavities at the ends of the sample. The conical platen was capable of reducing friction, but the deformation was still inhomogeneous [3]. Polosatkin [4] proposed using platens which rotated around an axis lying parallel to the axis of the cylindrical specimen. The die rotation approach reduced compression loads and also decreased the sidewall bulging effect with a more homogeneous deformation developed [5-7].

A number of workers have examined compression between rotating dies, using plasticine [5,6], or aluminium alloys [7]. These experimental observations verified that die rotation leads to a decrease in the compression load and the degree of bulging. Moreover, the tests made by the present authors [7] revealed that not all the torque power supplied by the rotating platen was transferred into twisting the bulk of the material, which led to a circumferential slippage between the rotating tool and the material.

At least two mechanisms were identified in [8] to account for the reduction in compressive load that accompanies die rotation: one involves the reduction in the interfacial frictional resistance at the radial direction, and the other is based on the introduction of an internal twist shear stress. Various analytical models have been developed to incorporate either one or two of these mechanisms. These include the standard stress analyses [9-11] and the upper bound approach [7,12-15]. Finite element analyses of compression with die rotation have also been carryout out in [5,6,14,16].

Circumferential slippage was not considered in these numerical simulations although a load reduction was observed when dies were rotated.

Experiments show that the degree of material twist is so low that it can safely be ignored at moderate compressive strains [7,8]. Therefore, the following stress analysis assumes that the only mechanism accounting for the compression load drop using rotating die is the diversion of friction stress vector due to the relative circumferential movement between the die and the material. The normal stress distribution over the tool/material interface can be established from this analysis and an estimate of load reduction limit is derived.

## 2. Slab Analysis



Figure 1: An infinitesimal element slice.

The compression speed of the axially driven platen is  $\dot{U}$  and the angular velocity of the rotating platen is  $\dot{\omega}_d$ . The material is assumed to be compressed without any twisting. In Figure 1, a material element slice is bounded on two sides by two radial planes passing through the symmetrical axis *y* and enclosing an angle  $d\theta$ , and by two concentric cylinders of radius *r* and *r* + *dr*. The height of slice is the same as the height *H* of the specimen. In compression without die rotation, the material point *P* on the top surface of the element has a relative velocity  $\dot{U}_r$  in the radial direction to the corresponding point *P'* initially adjacent to it on the die. Die rotation causes *P* to have an additional relative velocity component  $-\dot{U}_{\theta}$  in the circumferential direction. Thus, the shear stress,  $\mu p$ , due to friction on the material, acts on an angle of  $\phi$  to the circumferential direction (where  $\mu$  is the friction coefficient and *p* is the compression pressure).  $\phi$  is also equal to the acute angle between the direction of  $\dot{U}_{re}$  (the resultant relative velocity of point *P*) and  $-\dot{U}_{\theta}$ . If homogenous deformation is assumed, the radial velocity of point *P* may be expressed as:

$$\dot{U}_r = -\frac{1}{2}\frac{\dot{U}r}{H},\tag{1}$$

The velocity of P' is:

$$\dot{U}_{\theta} = \dot{\omega}_d r \tag{2}$$

So  $\phi$  can be determined by:

$$\tan\phi = \left|\frac{\dot{U}_r}{\dot{U}_\theta}\right| = \frac{1}{2}\frac{\dot{U}}{H\dot{\omega}_d}$$
(3)

The equilibrium equation of the element gives:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = -\frac{2\mu p \sin \phi}{H}$$
(4)

Assuming that the von Mises yield criteria is obeyed for the material, the yield equation for point *P* can be described as:

$$2(\sigma_r - p)^2 + 6(\mu p \cos \phi)^2 + 6(\mu p \sin \phi)^2 = 2\sigma_0^2$$
 (5a)

where  $\sigma_0$  is the yield stress from the tensile test.

Eq. (5a) can be reduced to:

$$\sigma_r = p - \sqrt{\sigma_0^2 - 3(\mu p)^2}$$
(5)

Inserting Eq. (5) into (4) gives:

$$\frac{dp}{dr} \left( 1 + \frac{3\mu^2 p}{\sqrt{\sigma_0^2 - 3(\mu p)^2}} \right) = -\frac{2\mu p \sin \phi}{H}$$
(6)

After integrating Eq. (6), *p* can be expressed by:

$$\ln p + 3\mu^2 \arcsin\left(\frac{\sqrt{3}\mu p}{\sigma_0}\right) = -\frac{2\mu r \sin\phi}{H} + C$$
(7)

where *C* is a constant that can be determined by the boundary condition:  $\sigma_{r=R} = 0$  (*R* is the outer radius of the sample). Setting  $\sigma_r = 0$  in Eq. (5) gives:

$$p = \frac{\sigma_0}{\sqrt{1+3\mu^2}} \quad (r = R) \tag{8}$$

Substituting Eq. (8) into Eq. (7) results in:

$$C = \ln\left(\frac{\sigma_0}{\sqrt{1+3\mu^2}}\right) + 3\mu^2 \arcsin\left(\frac{\sqrt{3}\mu}{\sqrt{1+3\mu^2}}\right) + \frac{2\mu R\sin\phi}{H}$$
(9)

Thus, Eq. (7) may be converted into:

$$\ln\left(\frac{\sqrt{1+3\mu^2}\,p}{\sigma_0}\right) + 3\mu^2 \arcsin\left(\frac{\sqrt{3}\,\mu p}{\sigma_0}\right) = 3\mu^2 \arcsin\left(\frac{\sqrt{3}\,\mu}{\sqrt{1+3\mu^2}}\right) + \frac{2\mu}{H}(R-r)\sin\phi \qquad (10)$$

The expression about *p* in Eq. (10) is in an implicit form and a closed form is difficult to find. However, the square root in the right-hand side of Eq. (5) can be removed by noting that  $\sigma_0 \ge \sqrt{3}\mu p$  and employing the following simplification:

$$\sqrt{a_1^2 - a_2^2} \approx 1.12a_1 - 0.7a_2$$
  $(a_1 \ge a_2 \ge 0)$  (11)

This linear approximation is the best linear fit over the range of  $a_1 \ge a_2 \ge 0$  by the least square fitting method. Eq. (5) can therefore be approximated by:

$$\sigma_r \approx (1.0 + 1.21\mu)p - 1.12\sigma_0$$
 (5b)

Consequently, Eq. (10) is reduced to:

$$p = \frac{\sigma_0}{\sqrt{1.0 + 3\mu^2}} \exp\left[\frac{2\mu}{(1.0 + 1.21\mu)H}(R - r)\sin\phi\right]$$
(12a)

Furthermore, if the shear stress in Eq. (5) is ignored, Eq. (10) can be reduced to:

$$p = \sigma_0 \exp\left[\frac{2\mu}{H}(R-r)\sin\phi\right]$$
(12b)

Finally, the compression load  $P_c$  according to Eqs (12a) and (12b) can be expressed by:

$$P_{c} = \int_{0}^{R} 2\pi r p \, dr = \frac{2\pi\sigma_{0}}{\sqrt{1+3\mu^{2}}} \left\{ \left[ \frac{(1.0+1.21\mu)H}{2\mu\sin\phi} \right]^{2} \exp\left[ \frac{2\mu R\sin\phi}{H(1.0+1.21\mu)} \right]^{2} - \frac{(1.0+1.21\mu)RH}{2\mu\sin\phi} - \left[ \frac{(1.0+1.21\mu)H}{2\mu\sin\phi} \right] \right\}$$
(13a)

and

$$P_{c} = 2\pi\sigma_{0} \left[ \left( \frac{H}{2\mu\sin\phi} \right)^{2} \exp\left( \frac{2\mu R\sin\phi}{H} \right) - \frac{RH}{2\mu\sin\phi} - \left( \frac{H}{2\mu\sin\phi} \right)^{2} \right]$$
(13b)

#### 3. Discussion

Eqs (13a) and (13b) relate the angular velocity of the die to the reduction of compression load. Figure 2 illustrates the distribution of compression pressure calculated by Eq. (13b) for a compression speed of 1.0 mm/sec, an angular velocity of 0.6 rad/sec, and a height and radius both of 1.0 mm. It can be seen that die rotation is predicted to decrease the compression load. In fact, the die rotation diverts the friction vector in Figure 1 and this effect causes a smaller effective friction coefficient in the radial direction, i.e.  $\mu_r = \mu \sin \phi$ . If  $\phi \rightarrow 0(\dot{\omega}_d \rightarrow \infty)$ , the compression pressure approaches the yield stress  $\sigma_0$ .

Figure 3 shows the effect of angular velocity of the die on the reduction in the compression load normalized by the yield stress using Eq. (13b). For these tests, the initial height and radius are 12 and 4 mm, the compression speed is 31 mm/min, and the instant height is 7.2 mm (corresponding to an engineering compressive strain of 40%). Also shown are experimental data obtained using an Aluminium alloy (yield stress of 250Mpa [8]). It can be seen that theoretically the efficiency of load reduction using tool rotation is less with increasing the angular velocity of the die. This effect agrees, qualitatively, with experiment. The discrepancy between them may be due to the ignoring of the material twisting in the analytical treatment. It is expected that the inclusion of the twist shear deformation is capable of decreasing the forming load further [8]. The experimental observation of increasing load reduction with increasing friction is also replicated by the model [17].

Furthermore, if the radial stress  $\sigma_r$  is neglected in the expression of the yield equation of Eq. (5a), the stressed state becomes plane stress and Eq. (5a) reduces to:

$$p = \frac{\sigma_0}{\sqrt{1+3\mu^2}} \tag{14}$$

It is clear that the compression pressure is less than the yield stress if the coulomb friction  $\mu$  is present. When  $\mu$  approaches its maximum value of  $1/\sqrt{3}$ , *p* attains a minimum, *p<sub>min</sub>*:

$$p_{min} = \frac{\sigma_0}{\sqrt{2}} \approx 0.707 \ \sigma_0 \tag{15}$$



Figure 2: Distribution of compression pressure.

Figure 3: Compression loads under various rotating speeds of the die.

0.8 0.9

Eq. (13b) Experime

Therefore, the compression pressure may be reduced to 70 percent of yield stress at the limit condition. This statement agrees with the experimental observation [17] in which the reduction percentage in compression load is found to be no more than 30% of the yield stress of Aluminium alloy (250Mpa).

### 4. Conclusion

In this conventional slab analysis, the only mechanism for the compression load drop with rotating die is diversion of friction stress vector due to the relative circumferential movement between the die and the material. Nevertheless, the expressions for compression pressure in Eqs (10) and (12a) do include the interfacial shear stress but they are only suitable for low friction conditions. The analytical result shows that the level of compression pressure can be lowered using die rotation. It also suggests that there is a limitation for the load reduction that is never larger than 30 percent of the yield limit.

#### References

- K. Lange. Handbook of metal forming, McGraw-Hill: New York, 1985.
- [1] [2] E. Siebel, and A. Pomp. Mitteilungen aus dem Kaiser-Wilhelm Institut fur Eisenforschung, 9 (8): 157, 1927
- T.C. Hsu, and A.J. Young. The Journal of Strain Analysis, 2 (2): 159-170, 1967. [3] [4] [5] [6]
- F.D. Polosatkin. Zavodskaya Laboratoriya, 7: 849-851, 1957.
- S. Xu, and Y. Fu. Journal of Material Processing Technology, 69 (1-3): 208-211, 1997. J.H. Park, Y.H. Kim, and Y.E. Jin. Journal of Materials Processing Technology, 111 (1-3): 103-106, 2001.
- X. Ma, M.R. Barnett, and Y.H. Kim. Key Engineering Materials, 233-236: 767-772, 2003.
- [8] [9] X. Ma, M.R. Barnett, and Y.H. Kim. International Journal of Mechanical Sciences, in press, 2003.
- O.A. Ganago, I.A. Bociarov, V.N. Subici, B.A. Stepanov, and A.V. Safonov. Izvestiia Vysshikh Uchebnykh Zavedenii: Mashinostroenie (2): 140-144, 1978.
- [10] O.A. Ganago, V.N. Subich, B.A. Stepanov, and A.V. Safonov. Izvestiia Vysshikh Uchebnykh Zavedenii: Mashinostroenie (6): 110-113, 1980.
- V.N. Subich. Izvestiia Vysshikh Uchebnykh Zavedenii: Mashinostroenie (10): 110-113, 1984. [11]
- A.N. Levanov, V.L. Kolmogorov, and S.P. Burkin. Contact friction in metalworking processes, Metallurgy Publishing House: Moscow, 1976. [12]
- K.A. Padmanabhan, R.A. Vasin, and F.U. Enikeev. Superplastic flow: Phenomenology and mechanics, Springer: Berlin, 2001. [13]
- Y.H. Kim, J.H. Park, and Y.E. Jin. Journal of Engineering Manufacture, Part B, Proceedings of the [14] Institution of Mechanical Engineers, 215 (6): 883-886, 2001
- [15] B. Avitzur, and J.Z. Pan. International Journal of Machine Tool Design and Research, 25 (3): 269-284, 1985.
- [16] K. Xue, Z. Wang, and Y. Lu. Journal of Materials Processing Technology, 69 (1-3): 148-151, 1997.
- [17] X. Ma. Compression and extrusion of metals using rotating dies. PhD Thesis, Deakin University: Geelong. 2003.