Modelling the Stress-Strain Behaviour for Aluminum Alloy AA6111

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Abstract

It is becoming increasingly important to have physically based predictive models for mechanical properties such as yield stress and work hardening behaviour. In this work, a yield stress model for heat treatments after the solution treatment has been developed in the internal state variable framework. The yield strength response is determined from the key microstructural parameters, i.e. the volume fraction and size of the strengthening phase. The work hardening behaviour as a function of the precipitate state has been considered in terms of the dislocation/precipitate interactions (i.e. whether the precipitates are shearable or not) and the appropriate flow stress superposition law for dislocation and precipitation hardening.

1. Introduction

There has recently been a resurgence of interest in examining 6000 series aluminum alloys for automotive and aerospace applications. Over the past several years, significant progress has been made in understanding the precipitation sequence and the effect of multi-step heat treatment on age hardening response. For example, detailed precipitation strengthening models have been developed [1-3] for the automotive alloy AA6111, and a careful consideration of the change in work hardening behaviour for different precipitate states [4] has been determined. The current work attempts to combine the models for precipitation strengthening and work hardening to produce a comprehensive model which describes the stress-strain behaviour up to the necking point for a variety of underaged, peak aged and overaged conditions in AA6111. This is of value from a number of perspectives including i) the potential use of the model as an input to finite elements models which are used to simulate metal forming or crash worthiness or ii) as a tool to aid alloy development.

The approach taken in the work follows the internal state variable framework where the key microstructural parameters, i.e. the precipitate size, volume fraction and size distribution, are explicitly used as input parameters. Work hardening is understood in terms of the competition between dislocation storage and dynamic recovery and how this balance is affected by the precipitate population. Finally, careful attention is paid to the question of how the various flow stress contributions should be summed to give the overall mechanical response.

2. Model Development

2.1 Yield Stress Model

The yield stress for a precipitation hardening alloy can be described by a summation of the intrinsic strengthening (i.e. lattice resistance and grain size strengthening), the solid solution contribution and the precipitation hardening contribution, i.e.

$$\sigma = \sigma_o + \sigma_{ss} + \sigma_{ppt} \tag{1}$$

The solid solution and precipitation hardening components are fundamentally linked by a mass balance of the alloying additions and are, thus, interdependent. The precipitation hardening contribution is a function of the average strength of precipitates as obstacles to dislocation motion, \overline{F} , and the spacing between these obstacles, L, and is given by:

$$\sigma_{ppt} = \frac{MF}{bL} \tag{2}$$

where *M* is the Taylor factor (3.06) and *b* is the magnitude of the Burgers vector (i.e. 0.286 nm for Al). In the case of AA6111, careful TEM examination has revealed that β'' and Q' precipitates coexist in various ratios during ageing at temperatures between 180 and 250 °C(see ref. [1,2]). In this work, the simplifying assumption will be made that the two precipitates can be treated as a single precipitation population. Further, since these precipitates form needles or laths parallel to the <100> direction, the average spacing on the (111) glide plane, *L*, can be determined as:

$$L = \left(\frac{2\pi}{f}\right)^{1/2} \overline{R} \tag{3}$$

where \overline{R} is the average equivalent radius and f is the total volume fraction of precipitates [1, 3]. The average obstacle strength is determined using the result of Deschamps and Brechet [5] which assumes: i) a linear dependency of precipitate strength on size for shearable precipitates; ii) a size independent strength for non-shearable precipitates and; iii) a Gaussian distribution of precipitate sizes. The result for average obstacle strength is then:

$$\overline{F} = kGb \left(\frac{K\Delta^2}{2} \left[\exp\left(-\frac{\overline{R}^2}{\Delta^2}\right) - \exp\left(-\frac{(R_c - \overline{R})^2}{\Delta^2}\right) \right] + \frac{K\Delta\overline{R}\sqrt{\pi}}{2} \left[\operatorname{erf}\left(\frac{\overline{R}}{\Delta}\right) + \operatorname{erf}\left(\frac{R_c - \overline{R}}{\Delta}\right) \right] \right) + kGbR_c \left(\frac{K\Delta\sqrt{\pi}}{2} \left[1 - \operatorname{erf}\left(\frac{R_c - \overline{R}}{\Delta}\right) \right] \right)$$
(4a)

where R_c is the critical radius for the transition from dislocation shearing to dislocation bypassing, Δ is the standard deviation of the precipitate radius distribution, *G* is the shear modulus of aluminum (27 GPa) and where *k* and *K* are given by:

$$k = \frac{b}{R_c}$$
 and $K = \frac{2}{\Delta\sqrt{\pi}\left(1 + erf\left(\overline{R} / \Delta\right)\right)}$ (4b)

Substituting equations (3) and (4) into equation (2) allows for a prediction of the precipitate strengthening contribution. The values for f, \overline{R} and Δ have been determined for a wide

range of ageing conditions (see Table 1) in a previous study using quantitative transmission electron microscopy. This leaves only a single unknown variable in the precipitation hardening model, the shearable/non-shearable transition radius. In this work, a good fit to the data was found with $R_c = 2.6$ nm. Finally, the solid solution contribution to strengthening, σ_{ss} , is estimated as:

$$\sigma_{ss} = \sigma_{sso} \left(1 - \alpha \frac{f}{f_{eq}} \right)^{2/3}$$
(5)

where σ_{sso} is the solid solution contribution for the solution treated material, f_{eq} is the equilibrium volume fraction of the precipitating phase (i.e. 0.019) and α is constant. The overall model results are rather insensitive to the value of α and this was chosen to be 1, i.e. when $f = f_{eq}$, $\sigma_{ss} = 0$

Table 1: Summary of data from quantitative TEM characterizing precipitate equivalent radius, volume fraction and size distribution [1,2]. Notes: i) Δ / \overline{R} is the ratio of the standard deviation of the size distribution to the mean radius and ii) the values marked with asteric are estimates.

Ageing Condition	volume fraction (β'' and Q')	mean equivalent radius / nm	Δ / \overline{R}
0.25 h @ 180 °C	0.002	1.2	0.3
0.5 h @ 180 °C	0.0036	1.2	0.2
1 h @ 180 °C	0.0063	1.4	0.2
7 h @ 180 °C	0.0072	1.8	0.2
60 days @ 180 °C	0.0072*	2.5-3.0	0.2
0.5 h @ 250 °C	0.012	4.5	0.3
7 days @ 250 °C	0.018 [*]	9.5 [*]	-

2.2 Work Hardening Model

The work hardening behaviour of a precipitation hardening alloy is a function of two main factors, i.e. i) the modification of dislocation storage by precipitates and ii) the appropriate superposition law for adding precipitate and dislocation contributions to the flow stress. The evolution of dislocation density with strain can be written in the Kocks/Mecking/Estrin framework [6,7] as:

$$\frac{\partial \rho}{\partial \varepsilon^{P}} = (k_1 \rho^{\frac{1}{2}} - k_2 \rho + k_D)$$
(6)

where ρ is the dislocation density, ε^{p} is the plastic strain, k_{1} is related to the storage of dislocation vis dislocation/dislocation interactions, k_{2} is the rate of dynamic recovery and k_{D} is an additional storage term due to dislocation/precipitate interactions. The flow stress contribution from dislocation hardening is given by:

$$\sigma_{\perp} = \alpha_{\perp} G b M \rho^{\frac{1}{2}}$$
⁽⁷⁾

where α_{\perp} is a constant of magnitude 0.3.

2.3 Shearable Precipitates

In the case of shearable precipitates, k_D is zero. As a result, the dislocation contribution to flow stress as a function of strain can be determined by integrating equation (6) and substituting into (7). This gives the well known Voce equation, i.e.

$$\sigma_{\perp} = \sigma_{\perp s} - \sigma_{\perp s} \exp\left(-\frac{\theta_{\perp o}}{\sigma_{\perp s}}\varepsilon^{P}\right)$$
(8)

where $\sigma_{\perp s}$ is the saturation stress and $\theta_{\perp o}$ is the initial work hardening rate. These two parameters can be related to k_1 and k_2 as:

$$\theta_{\perp o} = \frac{\alpha_{\perp} GbMk_1}{2} \tag{9a}$$

and

$$\sigma_{\perp s} = \frac{\alpha_{\perp} GbMk_1}{k_2}$$
(9b)

The values for k_1 and k_2 were determined by fitting the Voce equation to the plastic behaviour of the solution treated material. This gave values of $k_1 = 7.5 \times 10^8 \text{ m}^{-1}$ and $k_2 = 27$. In subsequent calculations, the rate of dynamic recovery is assumed to be independent of the precipitate population for both shearable and non-shearable precipitates.

The flow stress addition problem has been treated using a generalized addition law, i.e.

$$\sigma = \sigma_{ss} + \left(\sigma_{\perp}^{n} + \sigma_{ppt}^{n}\right)^{\frac{1}{n}}$$
(10)

when *n* is variable between 1 and 2. The physical basis of this approach relates to the relative density and strength of the different obstacles. For example, when the precipitates are weak obstacles (or alternatively small in radius) and they are summed with strong obstacles such as forest dislocations, the simulations of Foreman and Makin suggest one should sum the flow stress contributions, i.e. n = 1. On the other hand, for the case when the precipitates are strong (non-shearable), the density of obstacles (precipitates and forest dislocations) should add in linear manner, i.e. n = 2. In this work, an empirical approach has been taken by assuming that *n* varies linearly with the precipitate radius (or alternatively precipitate strength) from the smallest precipitate radius that was experimentally measured to the transition radius, i.e.

$$n = 0.13 + 0.72R \tag{11}$$

Equation (11) is valid for \overline{R} between 1.2 and 2.6 nm. For \overline{R} >2.6 nm, it is assumed that n = 2.

2.4 Non-Shearable Precipitates

In this case, it is assumed that the storage of dislocations in dominated by non-shearable precipitates such that in equation (6) $k_1 = 0$ and k_D describes the storage of dislocations due to precipitates [4]. The integration of equation (6) and substitution into equation (7) yields:

$$\sigma_{\perp} = \alpha GbM \left[\frac{k_D}{k_2} \left(1 - \exp\left(-k_2 \varepsilon^p\right) \right) \right]^{\frac{1}{2}}$$
(12)

where the value of k_D is geometrically related to the precipitate spacing as follows:

$$k_D = \alpha_D \frac{M}{bL} \tag{13}$$

 α_D is a constant and the precipitate spacing *L* is given by equation (3). In reference [4], a good fit to data was found when $\alpha_D = 0.3$ so that will be used here. Finally, since by

definition $\overline{R} > R_c$, a value of 2 is used for *n* in equation (8) when the final flow stress is calculated.

2.5 Prediction of Uniform Elongation

Having developed a work hardening model, it is now possible to also predict the extent of uniform elongation using the Considére criterion, i.e.

$$\sigma = \frac{d\sigma}{d\varepsilon} \tag{14}$$

In the present case, the derivative $\frac{d\sigma}{d\varepsilon}$, was determined by numerically differentiating the calculated stress-strain curve. Finally, given the magnitude of the uniform elongation, it would be straightforward to calculate the ultimate tensile stress if desired.

3. Results and Discussion

Figure 1 and 2 compare the results for the experimental and model predictions for stressstrain curves for the underaged and overaged samples, respectively. It can be observed that there is good agreement for both the yield stress and the work hardening behaviour for all cases. Figure 3 summarizes the comparison of model results and experimental results for yield stress and uniform elongation. It can be observed that over a wide range of conditions, the yield stress predictions are within ± 5 % of the experimental values while the predictions for uniform elongation also fall within this range with the exception of the two values for highly overaged samples with non-shearable precipitates where the deviation is slightly larger.

The advantage of the current approach is the physical basis of the model and the minimum number of adjustable parameters. The average radius, \overline{R} , the volume fraction, f, and width of the size distribution, Δ , have been directly measured. The values for k_1 and k_2 have a physical basis and are determined from work hardening behaviour of the solution treated sample. After the transition to non-shearable precipitates, the value of k_D is determined from the average precipitate spacing. The critical adjustable parameter is the transition radius, R_c , which once determined should be applicable to all alloys with similar precipitates. The weakest aspect of the model is the flow stress addition law where an empirical relation has been used, i.e. equations (8) and (9). This is an area which, in general, requires further investigation.



Figure 1: Comparison of experimental and model true stress vs. true plastic strain curves for underaged samples.



Figure 2: Comparison of experimental and model true stress vs. true plastic strain curves for overaged samples. Note: the samples aged for 0.5 hours and 7 days at 250 C are assumed to have non-shearable precipitates (see text).



Figure 3: Comparison of model predictions with experimental measurements of a) yield stress and b) uniform elongation. Dashed lines represent \pm 5 % deviation. Closed symbols and open symbols are for shearable and non-shearable precipitates, respectively.

4. Summary

A comprehensive model framework has been presented to describe the yield stress and work hardening behaviour for an industrially relevant aluminum alloy. Good agreement is observed between the model and experiments over a wide range of ageing conditions. The physical basis of the model offers the potential that the model can be expanded to a variety of alloys with different chemistries but where the strengthening precipitates are the same. This suggests the potential of the model for aiding alloy development.

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